Charge diffusion constant in hot and dense hadronic matter
- A Hadro-molecular-dynamic calculation -

N. Sasaki[*], O. Miyamura[†], S. Muroya[‡1)] and C. Nonaka[§]
Department of Physics, Hiroshima University,
Higashi-Hiroshima 739-8526, Japan
[1)] *Tokuyama Women's College, Tokuyama, 745-8511, Japan*

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Abstract

We evaluate charge diffusion constant of dense and hot hadronic matter based on the molecular dynamical method by using a hadronic collision generator which describes nuclear collisions at energies $10^{1\sim 2}$ GeV/A and satisfies detailed balance at low temperatures ($T \leq 200$ MeV). For the hot and dense hadronic matter of the temperature range, $T = 80 \sim 200$ MeV and baryon number density, $n_B = 0.16$ fm^{-3} ~ 0.32 fm^{-3}, charge diffusion constant D gradually increases from 0.5 fm$c$ to 2 fm$c$ with temperature and is almost independent of baryon number density. Based on the obtained diffusion constant we make simple discussions on the diffusion of charge fluctuation in ultrarelativistic nuclear collisions.

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[*]E-MAIL:nsasaki@butsuri.sci.hiroshima-u.ac.jp
[†]E-MAIL:miyamura@sci.hiroshima-u.ac.jp
[‡]E-MAIL:muroya@yukawa.kyoto-u.ac.jp
[§]E-MAIL:nonaka@butsuri.sci.hiroshima-u.ac.jp

1 Introduction

Searching for the quark gluon plasma (QGP) is one of the hottest topics in the recent high energy nuclear/particle physics [1]. Many kinds of phenomena have been proposed as candidates for the experimental signal to find the QGP. Charge fluctuation belongs to one of the most promising signals. Occurrence of Disoriented Chiral Condensation (DCC) at QCD phase transition would cause the large fluctuations of the ratio of the numbers of charged pions and neutral pions [2]. Difference of the fluctuation intensity between QGP phase and hadronic phase can work as a signal of the existence of QGP [3, 4, 5]. However, even if QCD phase transition takes place and the characteristic fluctuations of QGP or QCD phase transition are produced, such a fluctuation can be wiped out during hadronic era before the freeze-out. Whether the fluctuation which is caused at the phase transition or produced in the high temperature phase can survive in the hadronic era or not is the diffusion problem or the transport problem in the hot and dense hadronic matter. Macroscopic phenomenological equations, e.g., Navier-Stokes equation and diffusion equation, enable us to describe such phenomena in a simple manner. However, those kinds of phenomenological equations contain material constants, so called transport coefficients, and the dynamical calculation of the transport coefficients is a very important and difficult problem of statistical mechanics.

Interactions between hadrons are strong interaction which are believed to be described by QCD. However, in the hadronic energy region, perturbation does not work for QCD and no systematic way to treat is established. In a previous paper [6], we have reported a calculation of baryon number diffusion constant based on the relativistic collision event generator URASiMA (Ultra-Relativistic AA collision Simulator based on Multiple scattering Algorithm) which can reproduce hadronic spectra of nuclear collisions in the BNL-AGS and CERN-SPS. Usually collision event generators are so designed as to be suitable for the description of multiple production, and the detailed balance between the interactions is used to be paid only little attention [7]. As we discussed in the ref. [6], we improved the URASiMA to recover detailed balance in the hadronic time scale and we have succeeded to establish stationary states of interacting hadrons with fixed temperatures and fixed baryon number densities [9]. In this paper we evaluate charge diffusion constant in the hot and dense hadronic matter and investigate the diffusion of the charge fluctuation in the relativistic nuclear collisions.

2 Statistical ensembles

In order to prepare the statistical ensembles for the hot and dense hadronic state, we put hadrons in the box and updated with numerical code URASiMA with periodic boundary conditions. Recipes for the simulation are quite similar to the ordinary molecular dynamics. Molecular dynamics is one of the well established numerical methods in the statistical physics, however, it has been developed mainly for nonrelativistic system where particle number is conserved. The hot and dense hadronic system in which we have interest is a fully relativistic system, and the particle production and decay occur naturally. Therefore, the existence of stationary state itself is not apparent in this system.

URASiMA is originally designed as an event generator for the relativistic nuclear collisions based on the hadronic multichain model (MCM). In the high energy collision experiment, multiple production takes place and the system is thought to expand quickly. Therefore, usually, production processes play essential roles in the relativistic event generator but the reabsorption processes (reversal process of multiple production process) do not have been thought to be important, and sometimes have been neglected in the simulation code. However detailed balance of interactions is very important for statistical physics and naive application of collision event generator to the molecular dynamics in the box leads to the one-way conversion of the energy into particle production. As a result, Hagedorn type behavior appears, i.e., strange saturation of the temperature occurs [8, 10].

In order to recover the detailed balance, we have improved URASiMA to contain many resonances and changed the code so as to describe some of production processes occur through production and decay of the resonance. The reversal processes of those processes have been naturally taken into account. As a result, after initial thermalization time period of about 150 fm/c, detailed balances among interactions seem to be almost kept during the simulation. It is noted that such improvement does not spoil the descriptive power for ultrarelativistic nuclear collisions [9]. As we have already reported in ref. [6], slope parameter T of the energy distribution (Table 1),

$$\frac{dN}{d^3\boldsymbol{p}} = \frac{dN}{4\pi EpdE} = C\exp(-E/T),$$

became almost common value for all particles (fig. 1) and the population of particles became stationary (fig. 2). Therefore, we looked upon the system as the equilibrium state with temperature T and fixed baryon number density.

Table 1: Fitted parameter T of each particles at $t = 150$ fm/c. Baryon number density is normal nuclear density, $n_B = 0.156$ fm^{-3}.

ε_{tot} [GeV/fm^3]	N_{938} [MeV]	Δ_{1232} [MeV]	π [MeV]	ρ_{770} [MeV]
0.313	131 $\pm$ 5	122 $\pm$ 3	132$\pm$ 2	141 $\pm$ 5
0.625	170 $\pm$ 6	159 $\pm$ 5	163 $\pm$ 1	168 $\pm$ 2
0.938	181 $\pm$ 6	177 $\pm$ 6	187 $\pm$ 1	190 $\pm$ 2

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fig. 1
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fig. 2
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Running URASiMA many times with the same energy and the same number of baryons, N_B, in the box with volume V, and taking equilibrium configurations, we have prepared statistical ensembles of the state with temperature T and baryon number density N_B/V. Throughout this paper, we assumed iso-symmetry; the same number of protons and neutrons are put at initial time.

3 Diffusion constant in the linear response

According to Kubo's Linear Response Theory, diffusion constant D is obtained current (velocity) correlation [11]. Because of the relativistic property of the hot and dense hadronic state, we should use $\boldsymbol{\beta} = \boldsymbol{p}/E$ instead of usual $\boldsymbol{v}$,

$$D = \frac{1}{3}\int_0^\infty < \boldsymbol{\beta}(t) \cdot \boldsymbol{\beta}(t+t') > dt' c^2, \tag{1}$$

with c being the velocity of light. In the calculation of the charge diffusion, average is taken over all charged particles. When we evaluated baryon number diffusion constant D_B in our previous paper, we took average over only

baryons. If correlation of velocities damps exponentially,

$$< \boldsymbol{\beta}(t) \cdot \boldsymbol{\beta}(t+t') > \propto \exp(-\frac{t'}{\tau}) \tag{2}$$

with τ being relaxation time, we can rewrite eq. (1) with the simple form as,

$$\begin{aligned} D &= \frac{1}{3} < \boldsymbol{\beta}(t) \cdot \boldsymbol{\beta}(t) > c^2\tau, &(3)\\ &= \frac{1}{3} < \left(\frac{\boldsymbol{p}(t)}{E(t)}\right) \cdot \left(\frac{\boldsymbol{p}(t)}{E(t)}\right) > c^2\tau, &(4) \end{aligned}$$

with use of the relaxation time.

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fig. 3
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Figure 3 shows the correlations of the velocity of charged particles and exponential damping seems very good approximation. Diffusion constant obtained through eq. (3) is shown in fig. 4.

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fig. 4
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Diffusion constant increases with temperature and is almost independent of baryon number density. This result shows clear contrast against baryon number diffusion in our previous paper. Baryon number diffusion constant shows baryon number dependence and changes with temperature only weakly. Both calculations are almost the same but main contribution to each transport is different and we can understand fig. 4 as follows: In charge diffusion, main contribution comes from charged pions of which mass is just comparable to the temperature of the systems. On the other hand, the mass of the baryon is much larger than the temperature and baryonic system is almost nonrelativistic. The number of the baryon (baryonic charge carrier) is determined by the baryon number of the system, and even in very low temperature region, baryons must exist because of the fixed baryon number. In the lower temperature than pion mass, pion degrees of freedoms are frozen and electromagnetic charge and baryonic charge are both carried by baryons. At

temperature about pion mass, pion degrees of freedom start to be melted and start to contribute to the charge transfer. In the higher temperatures, pions dominate the charge transfer. As a result, in the temperature lower than pion mass, both diffusion constants are almost the same, but in the temperature higher than pion mass, diffusion constant of charge current gradually increases with temperature.

4 Charge fluctuation in ultrarelativistic nuclear collisions

Using the above obtained values, let us sketch the diffusion of the charge fluctuation in the relativistic nuclear collisions. In the simplest static picture, the solution of diffusion equation,

$$\frac{\partial}{\partial t} f(\boldsymbol{x}, t) = D\nabla^2 f(\boldsymbol{x}, t), \tag{5}$$

with the initial distribution,

$$f(\boldsymbol{x}, t_0) = \left(\sqrt{\frac{1}{2\pi R_0^2}}\right)^3 \mathrm{e}^{-\frac{(\boldsymbol{x}-\boldsymbol{x}_0)^2}{2R_0^2}} \tag{6}$$

is given by

$$f(\boldsymbol{x}, t) = \left(\sqrt{\frac{1}{\pi(2R_0^2 + 4D(t-t_0))}}\right)^3 \mathrm{e}^{-\frac{(\boldsymbol{x}-\boldsymbol{x}_0)^2}{2R_0^2+4D(t-t_0)}}. \tag{7}$$

Suppose that charge fluctuation with size R_0 produced at $t = t_0$, subjecting to eq. (5), the fluctuation diffuses and expands to $R(t) = \sqrt{R_0^2 + 2D(t-t_0)}$. Therefore, we may regard that at such time t that $2D(t-t_0) = R_0^2$, charge fluctuation almost disappears. If the charge fluctuation about $R_0 = 3$ fm at $T = T_C \cong 160$ MeV is produced, it survives only 2 fm/c during hadronic matter era. But if charge fluctuation is produced at lower temperature about 120 MeV (Super Cooled DCC), diffusion constant is about 0.7 fm$c$ and the fluctuation with initial size of about 3 fm will survive for 6 fm/c. The length of hadronic era and the chronological change of the temperature depend on the solution of the hydrodynamical model, but the latter case is promising to be observed experimentally.

In order to take account of expansion of the hadronic fluid, we must use relativistic hydrodynamical model. In the relativistic hydrodynamical model, charged current J^μ is given as,

$$J^\mu = f(\boldsymbol{x}, t)U^\mu + J^\mu_d, \tag{8}$$

with U^μ being local four velocity [12, 13]. $f(\boldsymbol{x}, t)$ is charge density on the local rest frame, $f(\boldsymbol{x}, t) = U^\mu J_\mu$. The diffusion current, J^μ_d is given by relativistic extension of the Fick's law[1]

$$J^\mu_d = \frac{D}{c}\Delta^\mu{}_\nu \partial^\nu f(\boldsymbol{x}, t), \tag{9}$$

where $\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$ is space-like projection operator orthogonal to U^μ. Putting above J^μ into the continuity equation, $\partial_\mu J^\mu = 0$, we can obtain the relativistic diffusion equation,

$$U^\mu \partial_\mu f(\boldsymbol{x}, t) + f(\boldsymbol{x}, t)\partial_\mu U^\mu + \frac{D}{c}\partial_\mu \Delta^\mu{}_\nu \partial^\nu f(\boldsymbol{x}, t) = 0. \tag{10}$$

In the case of 1+1 dimensional scaling expansion, particular solution of the local four velocity is given as $U^\mu = \frac{x^\mu}{c\tau}$ with light-like coordinate, $ct = c\tau\cosh\eta$, $z = c\tau\sinh\eta$ [14], and diffusion equation (10) is rewritten as,

$$\frac{\partial}{\partial\tau} f(\eta, \tau) + \frac{1}{\tau} f(\eta, \tau) = \frac{D}{c^2\tau^2}\frac{\partial^2}{\partial\eta^2} f(\eta, \tau). \tag{11}$$

The solution of eq. (11) with initial condition, $f(\eta, \tau_0) = \sqrt{\frac{1}{2\pi R_0^2}}\exp(-\frac{\eta^2}{2R_0^2}) f_0$, is given by

$$f(\eta, \tau) = \frac{\tau_0}{\tau}\sqrt{\frac{1}{2\pi R(\tau)^2}} \mathrm{e}^{-\frac{\eta^2}{2R(\tau)^2}} f_0 \tag{12}$$

with $R(\tau)^2$ being [2],

$$R(\tau)^2 = R_0^2 + 2\frac{D}{c^2\tau\tau_0}(\tau - \tau_0). \tag{13}$$

The first factor in eq. (12), $\frac{1}{\tau}$, is the result of systematic expansion of the scaling solution; the region with fixed $\Delta\eta$ expands as $\tau\Delta\eta$. This rapid expansion also dominates the evolution of the fluctuation size, $R(\tau)^2$; the region with fixed $\Delta\eta$ expands with τ but the diffusion effect subject to eqs. (5) or (

[1]In the relativistic notation, $x^0 = ct$, and $\frac{\partial}{\partial x^0} = \frac{\partial}{c\partial t}$.

[2]Note that this is the difussion in the η direction and R is the size in the η coordinate.

10) is the expansion with only $\sqrt{2D\tau}$. Therefore, in η coordinate, the effect of the diffusion becomes smaller and smaller with time, $\frac{D}{c^2\tau\tau_0}$. According to our result (fig. 4), diffusion constant in the hadronic matter is at most 2 fm$c$ and the diffusion term in eq. (13) becomes small enough already at τ_0= several fm. As a result, the fluctuation in η will be deformed only little by the diffusion effect in the hadronic era.

5 Concluding remarks

We evaluated charge diffusion constant of hot and dense hadronic matter based on the relativistic collision event generator URASiMA. Obtained charge diffusion constants increase from 0.5 fm$c$ to 2 fm$c$ in the temperature range between 80 MeV and 200 MeV. Based on the obtained diffusion constant, we made rough sketches of the diffusion of charge fluctuation in the ultrarelativistic nuclear collisions. For more improved discussion, we need to solve the hydrodynamical equation coupled with the charge current conservation equation [15].

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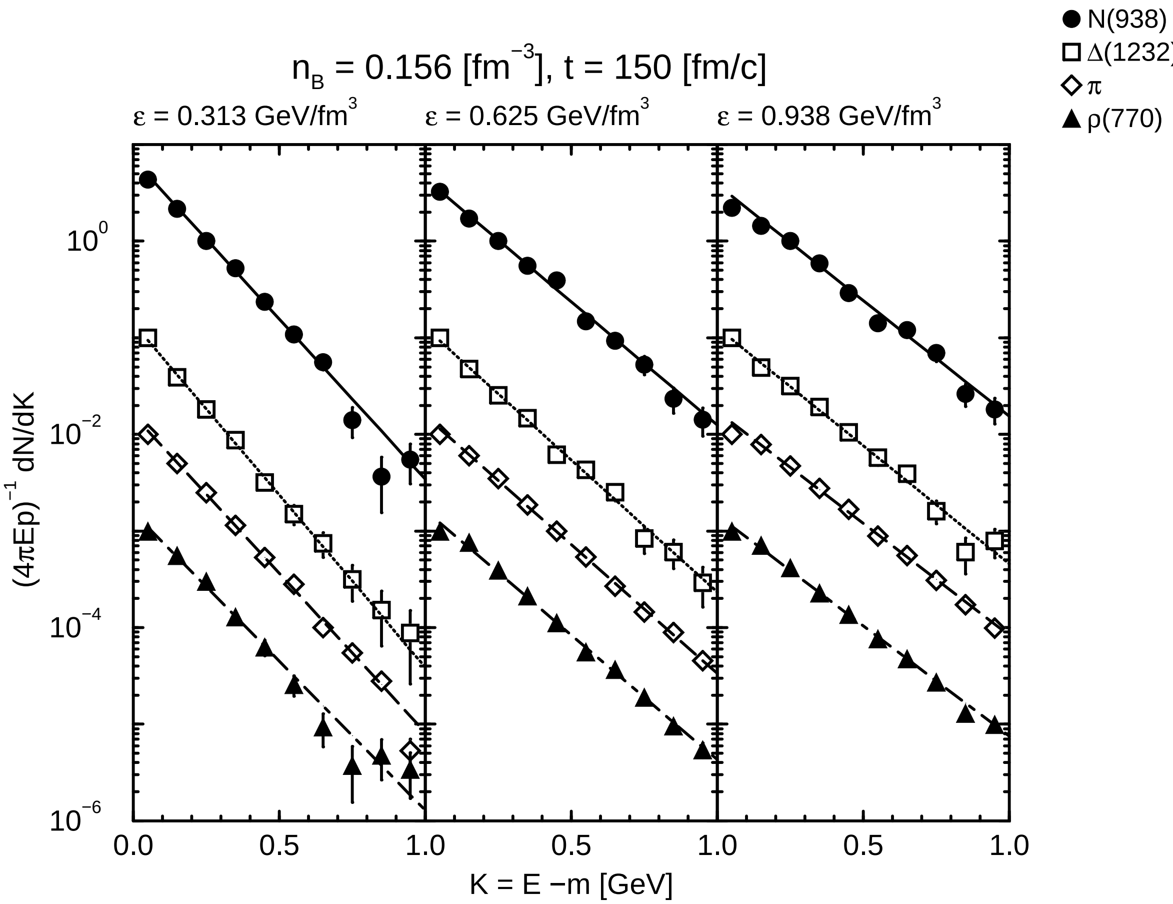

Figure 1: Energy distributions of particles at $t = 150$ fm/c. Lines stand for the results of fitting with thermal distribution. Normalizations of the data are arbitrary.

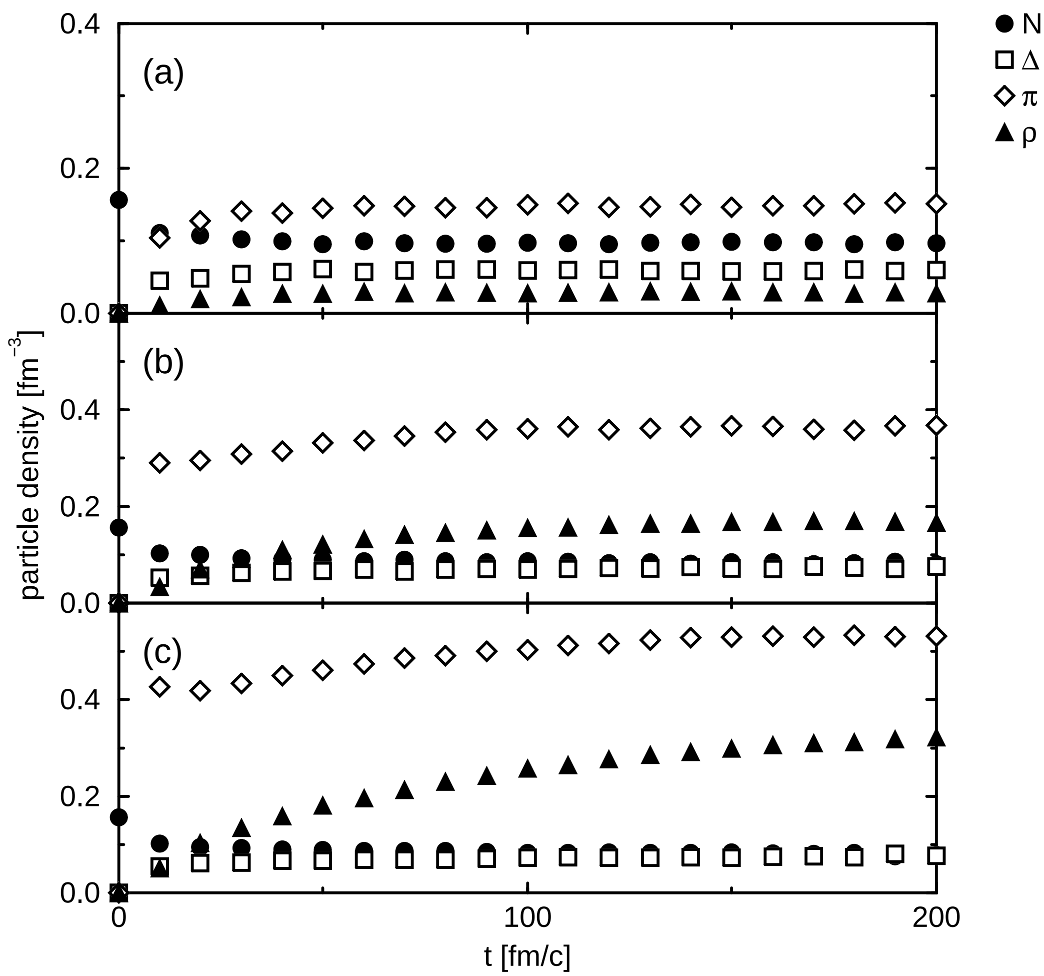

Figure 2: Particle density of time. The population of the particles in later than t = 150 fm/c seem to be stationary. In each case, baryon number density is $n_B = 0.156$ fm^{-3}, and energy density for (a) is 0.313 GeV/fm^3, (b) is 0.625 GeV/fm^3 and (c) is 0.938 GeV/fm^3, respectively.

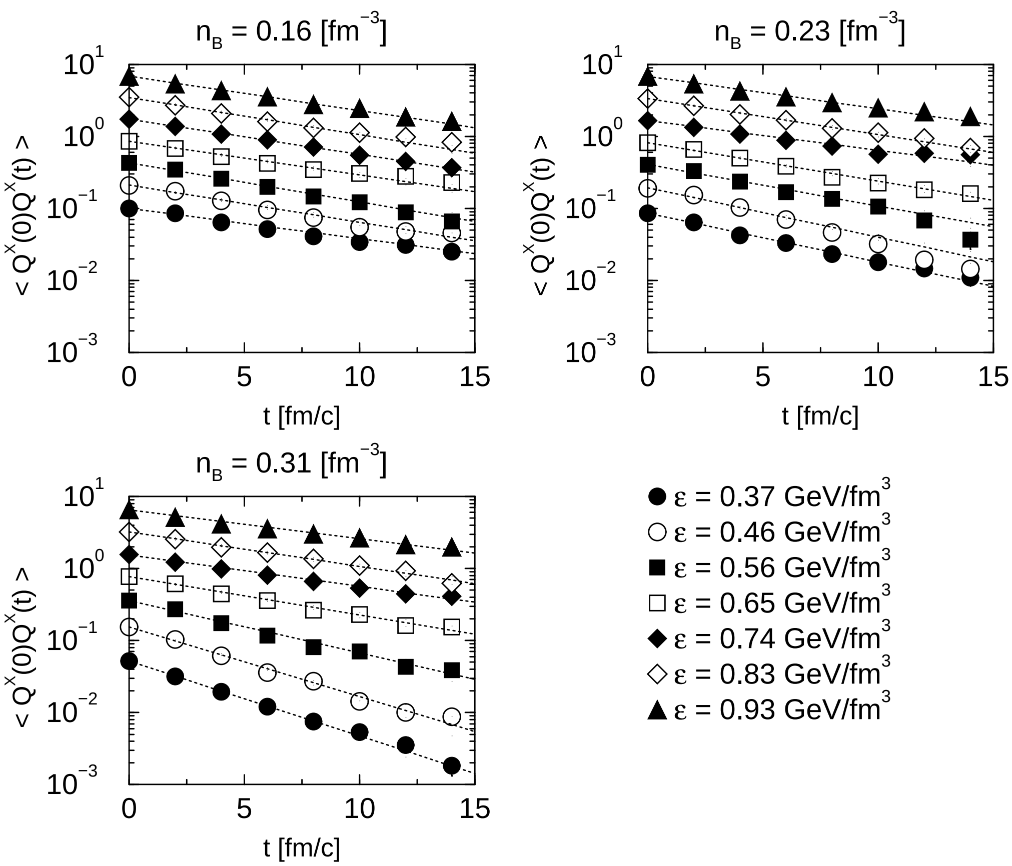

Figure 3: Velocity correlation of the charged particles as a function of time. Lines correspond to the fitted results by exponential function eq. (2). Normalizations of the data are arbitrary.

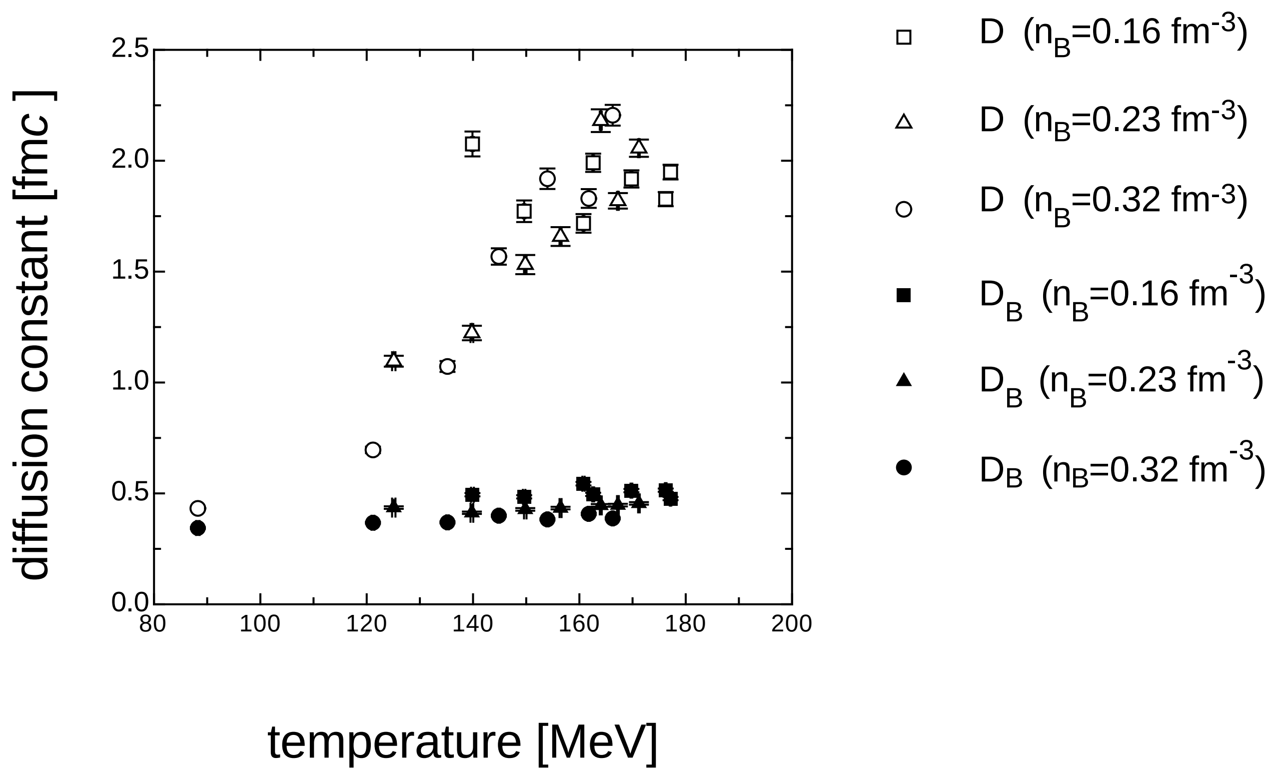

Figure 4: Diffusion constant D and baryon diffusion constant D_B.